How to Calculate Streamer Size

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When I first saw that the 2010 Team America Rocketry Challenge involved recovering - at a minimum - the payload section containing the egg and the altimeter by streamer, I thought “Holy Cow! How are they going to do that?” After I got over my initial shock, I set to work trying to create a trial design in RockSim version 9.0, but was frustrated by the program’s refusal to calculate the descents of the payload section and the sustainer separately (Note: I did eventually figure out how to “trick” RockSim into doing the right thing, but that was much later[1]). A search of the Internet brought no joy, as I could not find a streamer descent rate calculator anywhere; the realization gradually dawned that I was going to have to work this out myself.

It turns out that it is not so hard, actually. All we have to do is pay a little attention to Sir Issac Newton and his 1st Law of Motion, which states:

An object at rest tends to stay at rest and an object in motion tends to stay in motion with the same speed and in the same direction unless acted upon by an unbalanced force.

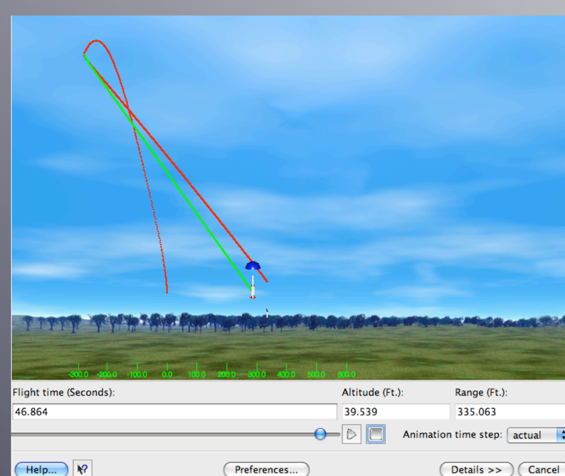
How does this apply to a payload section or rocket falling slowly (hopefully!) to the ground attached to a streamer? Ignore the business about an object at rest, and consider the

How to make RockSim compute separate descent tracks (works with versions 8 and 9).

1. Create a two stage rocket, with the payload section (and attached streamer) as the sustainer, and the rest of the rocket as the booster. In order to do this, the payload section tube must be designated as a motor mount, even though it will not contain a motor.

2. In the "Prepare for launch" -> "Flight events" section, select "Deploy at altitude" for the sustainer (payload section) and type in 800 (or whatever) in the altitude column. Then select "Deploy at stage separation" for the booster.

3. When you run the simulation, you will now have separate descent tracks for the payload section and the rocket.



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[1] It turns out that Tim Van Milligan, the developer of RockSim, addressed this in a video posted a few years back. Check out TARC Tutorial 3 at http://www.apogeerockets.com/RockSim_tutorials.asp

remaining words as they apply to streamer recovery. We want the payload section or rocket to fall at a slow, steady speed, right? If that's the case, the 1st Law tells us there can be no unbalanced forces.

So what are the forces acting on a rocket or payload section descending by a streamer? There is one that's obvious - the force of gravity pulling it down towards the ground - which is equal to the combined weight of the rocket and streamer. Any others? Yep - the force due to air resistance, which will act opposite gravity, trying to slow the rocket's fall. There also may be a force due to the wind pushing on the rocket, but let's keep things simple, and consider the case with no wind. Thus, if the rocket is to fall at a steady speed, the force of gravity (the weight) must be balanced by the force of air resistance (called drag). Put in mathematical terms,

$$\text{Drag force} = \text{Weight}$$

So how do we calculate the force of drag on the rocket/streamer combination? There is a simple expression that is found in the college physics books, which states that the drag force is proportional to the area of the rocket plus the area of the streamer, the density of the air, and the square of the speed at which it is falling. In equation form,

$$\text{Drag Force} = \frac{1}{2} C_D A \rho v^2$$

What do these symbols mean? Well, A is the area of the rocket plus the area of the streamer, ρ is the density of the air, and v is the fall speed. The symbol C_D stands for something called the drag coefficient, which is a number that indicates how "draggy" the object is. For example, a typical model rocket has a drag coefficient of about 0.75, and a ping pong ball has a C_D of roughly 1.2. Drag coefficients are generally measured using wind tunnels, as they are <u>extremely</u> difficult to calculate.

Newton's 1st Law says that the drag force must equal the weight if the rocket is to fall at constant speed, so

$$\frac{1}{2} C_D A \rho v^2 = W$$

where W stands for the weight of the rocket plus that of the streamer.

But how does all this math help me figure out the size of the streamer? Take a look at the equation above... We can measure or get from Rocksim the weight, right? And we can look up on the Internet (or in a book, if you're an old geezer like me), the density of air. That leaves the drag coefficient (C_D), the area of the rocket plus that of the streamer (A), and the fall speed (v).

It turns out that we know the fall speed, or at least can calculate it. The TARC 2010 competition has a target altitude of 825 feet and a duration target of 40 to 45 seconds. If we assume that the streamer is deployed right at 825 feet, and that we desire a total flight time of 43 seconds, then we want the rocket or payload section fall at a speed of

$$v = \frac{825\,\text{feet}}{35\,\text{seconds}} = 23.6\,\text{feet per second}$$

You will see that I used 35 seconds instead of 43 in the above calculation. That's because 43 seconds is the total time from launch until landing, and I took away about 8 seconds (your time may be much different) to allow the rocket enough time to reach apogee. All I am interested in here is the falling part - not the part where the rocket is going up.

Fortunately for us, the drag coefficient of streamers have been measured by the rocketeers of the past, and the C_D varies from 0.14 for a nice shiny mylar streamer to about 0.4 for a folded (like an accordion) strip of tracing paper[2]. The mylar is thin and light, but it cannot be folded like more draggy tracing paper streamer, which is heavier. Both types are easily available, and the choice of which to use is yours. For the example here, I will use the mylar, as, being less draggy, it will require a bigger streamer in terms of size.

We now know everything but the area of the streamer plus that of the rocket, and guess what? The area of the streamer tells us its size, which is what we are after. Studies have shown that the best streamers are skinny rectangles in which the length is 10 times the width. Since the area of a rectangle is length times width, then the area of the streamer is

$$A_{streamer} = \text{length x width} = 10t \times t = 10t^2$$

where I have used the symbol t to represent width, to avoid confusion with the weight W. We also can write that

$$A = A_{streamer} + A_{rocket} = 10t^2 + A_{rocket}$$

The area of the rocket is roughly its length multiplied by its diameter (ignoring the fins and the pointy shape of the nose cone). However, an obvious TARC strategy would be to bring just the payload section containing the egg and the altimeter down by the streamer[3], and let the rest of the rocket come down by parachute. That way gives us the smallest amount of weight for the streamer to slow down. As you may guess, we are

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[2] These drag coefficients are approximate, intended to provide a very rough starting point. The real world may be different, so flight testing becomes extremely important.

[3] One could also see a strategy in which a lightweight rocket remains attached to the payload section and helps to slow the fall by assuming a high drag orientation. Put some careful thought into design.

going to be dealing with a pretty big streamer (or streamers), so we can ignore the area of the payload section, and let A be equal to the area of the streamer. In this case,

$$A = A_{streamer} = 10t^2$$

Now we know everything but the width of the streamer, t, and once we know that we can get the streamer length by simply multiplying the width by 10. All that's left is to do a little algebra, and solve the equation

$$\frac{1}{2} C_D A \rho v^2 = W$$

for the area, A. Once this is done, we get

$$A = \frac{2W}{C_D \rho v^2}$$

Putting in the numbers:

$$A = \frac{2W}{(0.14)(0.0023284)(23.6)^2} = 11.016W \text{ square feet}$$

where W is the total weight in pounds. If we want to use ounces for the weight, and get the area in square inches, then the formula becomes

$$A = 99.14\ W \text{ square inches}$$

where W is in ounces. Once we know A, then we can get the width of the streamer from

$$A = 10t^2, \text{ or } t = \sqrt{\frac{A}{10}}$$

The length of the streamer is simply 10 times the width, or 10t.

This would be the end of the story, except that I lied a bit earlier. Remember when I said we could get the total weight by measuring it or from Rocksim? That's certainly true for the payload section, but how can I measure or calculate the weight of something (the streamer) I haven't designed yet? So how are we going to get the weight of the streamer before we know how big it has to be?

The simple answer is we can't. But what we can do is measure or calculate the weight of the payload section, use this as W, and calculate a streamer size. Once we know the size of the streamer, we can calculate its weight (assuming the mylar is 1 mil or 0.001“ thick) from

$$W_{mylar} = 0.00074152\ A$$

where W_{mylar} is the streamer weight in ounces, and A is the streamer area in square inches. The total weight, W, is then

$$W = W_{mylar} + W_{payload}$$

We then use this W to recalculate the area of the streamer, then get a new streamer weight, and repeat the process until there is very little difference between the streamer areas.

An example might help - let’s say that our payload section, with egg and altimeter, weighs 4 ounces. Then our first guess for the streamer size is

$$A = 99.14 \times 4 = 396.56 \text{ square inches}$$

The weight of this streamer is

$$W_{mylar} = 0.00074152 \times 396.56 = 0.294 \text{ ounces}$$

So our new weight is

$$W = 4 + 0.294 = 4.294 \text{ ounces}$$

which gives a new streamer size of

$$A = 99.14 \times 4.294 = 425.70 \text{ square inches}$$

which has a weight of

$$W_{mylar} = 0.00074152 \times 425.70 = 0.316 \text{ ounces}$$

Our new total weight is

$$W = 4 + 0.316 = 4.316 \text{ ounces}$$

which gives a streamer area of

$$A = 99.14 \times 4.316 = 427.90 \text{ square inches}$$

This is not very much different from 425.7 square inches, so we stop calculating, and declare our streamer will have an area of about 428 square inches. Therefore the dimensions of our streamer are

$$width = t = \sqrt{\frac{428}{10}} = 6.54\ \text{inches}$$

and so length = 10t = 65.4 inches.